5.5.2 Konvoluutio

Ensin (tulon muunnos):

$$\mathcal{F}\{u(t)v(t)\} = \int_{-\infty}^{\infty} u(t)\, v(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} U(jy)\, e^{jyt}\, dy \right] v(t)\, e^{-j\omega t}\, dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(jy)\, e^{jyt}\, v(t)\, e^{-j\omega t}\, dy\, dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(jy)\, e^{jyt}\, v(t)\, e^{-j\omega t}\, dt\, dy$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} U(jy) \left[\int_{-\infty}^{\infty} v(t)\, e^{-j(\omega - y)t}\, dt \right] dy$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} U(jy)\, V(j(\omega - y))\, dy$$

$$= \frac{1}{2\pi} U(j\omega) * V(j\omega)$$

Sitten vastaavasti (konvoluution muunnos):

$$\mathcal{F}\{u(t)*v(t)\} = \int_{-\infty}^{\infty} \left[u(t)*v(t)\right] e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} u(y)v(t-y)\; dy \right] e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(y)v(t-y)\; e^{-j\omega t}\, dy\; dt$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(y)v(t-y)\; e^{-j\omega t}\, dt\; dy$$

$$= \int_{-\infty}^{\infty} u(y) \left[\int_{-\infty}^{\infty} v(t-y)\, e^{-j\omega t}\, dt \right] dy$$

$$= \int_{-\infty}^{\infty} u(y) \left[\int_{-\infty}^{\infty} v(x)\, e^{-j\omega(x+y)}\, dx \right] dy$$

$$= \int_{-\infty}^{\infty} u(y)\, e^{-j\omega y} \left[\int_{-\infty}^{\infty} v(x)\, e^{-j\omega x}\, dx \right] dy$$

$$= \int_{-\infty}^{\infty} u(y)\, e^{-j\omega y} \left[V(j\omega)\right] dy = V(j\omega)U(j\omega)$$