

Teoreema 4.6 Parsevalin lause

Kertolaskulauseen tapaus $f(t) = g(t)$:

$$\frac{1}{T} \int_d^{d+T} f(t)^2 dt = \sum_{n=-\infty}^{\infty} c_n c_n^* = \sum_{n=-\infty}^{\infty} |c_n|^2$$

Entä reaalikertoimilla?

$$\begin{aligned} \frac{1}{T} \int_d^{d+T} f(t)^2 dt &= \\ &= c_0 c_0^* + \sum_{n=1}^{\infty} c_n c_n^* + \sum_{n=-1}^{-\infty} c_n c_n^* = \\ &= c_0 c_0^* + \sum_{n=1}^{\infty} c_n c_n^* + \sum_{n=1}^{\infty} c_n^* c_n = \\ &= c_0 c_0^* + 2 \sum_{n=1}^{\infty} c_n c_n^* = \end{aligned}$$

$$= \frac{1}{4} a_0^2 + 2 \sum_{n=1}^{\infty} \frac{a_n - jb_n}{2} \frac{a_n + jb_n}{2} =$$

joten

$$\frac{1}{T} \int_d^{d+T} f(t)^2 dt = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$