



MAT-33500 Differentiaaliyhtälöt
MAT-33506 Differential equations
Problem class 1 (period III week 4/2012)

1. Find the general solution of the logistic differential equation with constant harvesting:

$$x' = x(1 - x) - h$$

for all values of the parameter $h > 0$.

2. Find the eigenvalues and -vectors of the following matrices:

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \quad \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$$

3. Find the general solutions of the following linear systems:

$$X' = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} X \quad X' = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} X \quad X' = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} X$$

What is special about the last one?

4. Show that the two vectors $V = (v_1, v_2)$ and $W = (w_1, w_2)$ are linearly independent if and only if

$$\det \begin{bmatrix} v_1 & w_1 \\ v_2 & w_2 \end{bmatrix} \neq 0.$$

5. Consider the matrix

$$A = \begin{bmatrix} a & 1 \\ 0 & 1 \end{bmatrix}.$$

Find a value a_0 for the parameter a such that A has a repeated eigenvalue (i.e., two similar eigenvalues). What happens to the eigenvectors when $a \rightarrow a_0$?

6. By converting the harmonic oscillator equation

$$x'' + bx' + kx = 0$$

to a linear first-order system, find all values of b, k for which the system has real, distinct eigenvalues. Find the general solution of the system in these cases.