TAMPEREEN TEKNILLINEN YLIOPISTO
Matematiikan laitos

MAT-33500 Differentiaaliyhtälöt
MAT-33506 Differential equations
Problem class 2 (period III week 5/2012)

1. Find a 2×2 matrix A such that the curve

$$X(t) = \begin{bmatrix} e^{2t} - e^{-t} \\ e^{2t} + 2e^{-t} \end{bmatrix}$$

is a solution of the equation $X' = AX$.

2. Solve the (initial value) problem

$$X' = AX, \qquad X(0) = (3, -1) \qquad A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

and sketch the phase portrait of the system.

3. Consider the system $X' = AX$ with

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Find the eigenvalues and -vectors of A and determine a matrix T such that $T^{-1}AT$ is in a canonical form. Then determine the general solutions of the systems $X' = AX$ and $Y' = T^{-1}ATY$, and sketch the phase portraits.

4. As above, but with

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}.$$

5. As above, but with

$$A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}.$$

6. Consider the general harmonic oscillator (mass$= 1$)

$$X' = \begin{bmatrix} 0 & 1 \\ -k & -b \end{bmatrix} X,$$

with $b \geq 0$ and $k > 0$. With which values of k, b does the system have i) complex, ii) repeated, or iii) real (and distinct) eigenvalues? Determine the general solution for the case of complex eigenvalues $\alpha \pm i\beta$, and describe the position and velocity of the mass (as function of time) when $b > 0$ and the initial position is $x(0) = 1$ (and initially at rest).