TAMPEREEN TEKNILLINEN YLIOPISTO
Matematiikan laitos

MAT-33500 Differentiaaliyhtälöt
MAT-33506 Differential equations
Problem class 3 (period III week 6/2012)

1. Compute e^A using $e^{T^{-1}AT}$, when

$$A = \begin{bmatrix} 5 & -6 \\ 3 & -4 \end{bmatrix}.$$

What are the eigenvalues of e^A?

2. By using the formulas

$$\exp \begin{bmatrix} \lambda & 0 \\ 0 & \mu \end{bmatrix} = \begin{bmatrix} e^\lambda & 0 \\ 0 & e^\mu \end{bmatrix}, \qquad \exp \begin{bmatrix} 0 & \theta \\ \theta & 0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix},$$

and the rule $\exp(A + B) = \exp(A)\exp(B)$ (check that it is applicable!), compute

$$\exp\left(t \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}\right).$$

3. Let

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

Show that $A^n = 0$, and use this to compute $\exp(tA)$.

4. Let

$$A = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}.$$

Using the results above and $\exp(A+B) = \exp(A)\exp(B)$ (again, check that it is applicable!), compute $\exp(tA)$ (i.e., do not compute it in the same way as in the lectures).

5. When computing the eigenvalues of a 2×2-matrix, we solve a quadratic equation. In other words, we solve $f(\lambda) = 0$ when $f(\lambda)$ is parabolic. Depending on how $f(\lambda)$ is positioned with respect to $f = 0$ axis (in the (λ, f)-plane), we thus have either two distinct real roots for λ, or two complex-valued roots (complex conjugates of each other), or one real-valued double root. What is the corresponding classification for the eigenvalues of 3×3-matrices? Sketch graphs for the different cases.

6. The previous problem for 4×4 matrices.