

MAT-33500/6 Differentiaaliyhtälöt/Differential equations
Problem class 4 (period III week 7/2012)

1. a. Solve the nonautonomous differential equation

$$x' - 4x - \cos t = 0.$$

- b. Solve the nonhomogeneous system

$$x' = y, \quad y' = 2 - x.$$

2. Compute

$$\exp\left(t \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}\right)$$

by splitting the matrix into a diagonal and a nondiagonal part and then summing these.

3. Let

$$A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix}.$$

Find the general solution of the system $X' = AX$.

4. Find a function $s : \mathbb{R} \rightarrow \mathbb{R}$ that fulfills the scalar equation

$$s''' - s'' + 4s' - 4s = 0$$

with the initial values $s(0) = 1$, $s'(0) = -1$, $s''(0) = 1$.

5. Find the general solution of the nonautonomous system

$$x' = -y, \quad y' = x + t.$$

6. Solve the system $X' = AX$, when

a.

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

b.

$$A = \begin{bmatrix} 0 & 0 & a \\ 0 & b & 0 \\ a & 0 & 0 \end{bmatrix}.$$

Sketch the regions in the ab -plane where this system has different types of phase portraits.