MAT-33500/6 Differentiaaliyhtälöt/Differential equations
Problem class 5 (period III week 8/2012)

1. Find the general solution of the nonautonomous differential system

$$X' = A(t)X, \quad A(t) = \begin{bmatrix} 1 & 0 \\ \cos t & 1 \end{bmatrix}$$

by solving an autonomous and a nonautonomous equation (for which you can use the solution formula by integration).

2. Consider the nonlinear system

$$x' = x^2 - 2xy + x, \qquad y' = 3xy - y^2.$$

Find the equilibrium points, examine their stabilities, and sketch the phase portrait (with some estimated solution curves).

3. Describe the solutions of the nonlinear differential equation

$$x'' + 2x^3 - x = 0$$

by transforming it to a system for which the solution curves can be written in the form $v = v(x)$ by separation of variables (i.e., the phase portrait can be plotted without having to solve $v(t), x(t)$ explicitly). Sketch the phase portrait (including the equilibrium points).

4. Derive the Taylor expansion of the function $\sin 2t$ by using Picard iteration for a 2-part system corresponding to the differential equation

$$x'' = -4x, \quad x(0) = 0, \quad x'(0) = 2.$$

5. Consider the nonlinear system

$$x' = x(x^2 + y^2), \quad y' = y(x^2 + y^2).$$

Find the equilibrium points and describe the behaviour of the linearized system. Sketch the phase portrait of the nonlinear system. Is the behaviour of the linearized system similar to that of the nonlinear one near equilibrium points?

6. As above, but for the nonlinear system

$$x' = x + y^2, \quad y' = 2y.$$