MAT-51327 Numerical methods for partial differential equations Exercise 2
Samuli Siltanen and Juho Linna 20.2.2009

1. Define the convolution $f * g$ of test functions $f, g \in C_0^\infty(\mathbb{R}^n)$ by

$$(f * g)(x) = \int_{\mathbb{R}^n} f(x-y)g(y)dy.$$

Denote $\partial_j h := \frac{\partial}{\partial x_j} h(x)$.

(a) Show that $\partial_j (f * g) = (\partial_j f) * g = f * (\partial_j g)$.

(b) Show that $\delta * g = g$.

(c) Show that $\widehat{(f * g)}(\xi) = \widehat{f}(\xi)\widehat{g}(\xi)$, where $\widehat{\cdot}$ denotes Fourier transform.

2. Prove that, in the sense of distributions,

$$\overline{\partial}\left(\frac{1}{\pi(z_1 + iz_2)}\right) = \delta_0(z),$$

where $\overline{\partial} = (\partial/\partial z_1 + i\partial/\partial z_2)/2$. Hint: use Green's formula.

3. Use Matlab to solve equation

$$-\frac{\partial^2}{\partial x^2} u(x) = f(x), \qquad 0 < x < 1,$$

numerically using variational formulation and the Finite Element Method. Here f is given by

$$f(x) = \begin{cases} 1 \text{ for } \frac{1}{4} < x < \frac{3}{4}, \\ 0 \text{ otherwise.} \end{cases}$$

Follow the explanation in section 5.1 of the textbook and build the load vector b and stiffness matrix A yourself by numerical integration.