Mathematics makes the unseen visible (and vice versa)

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What are inverse problems?

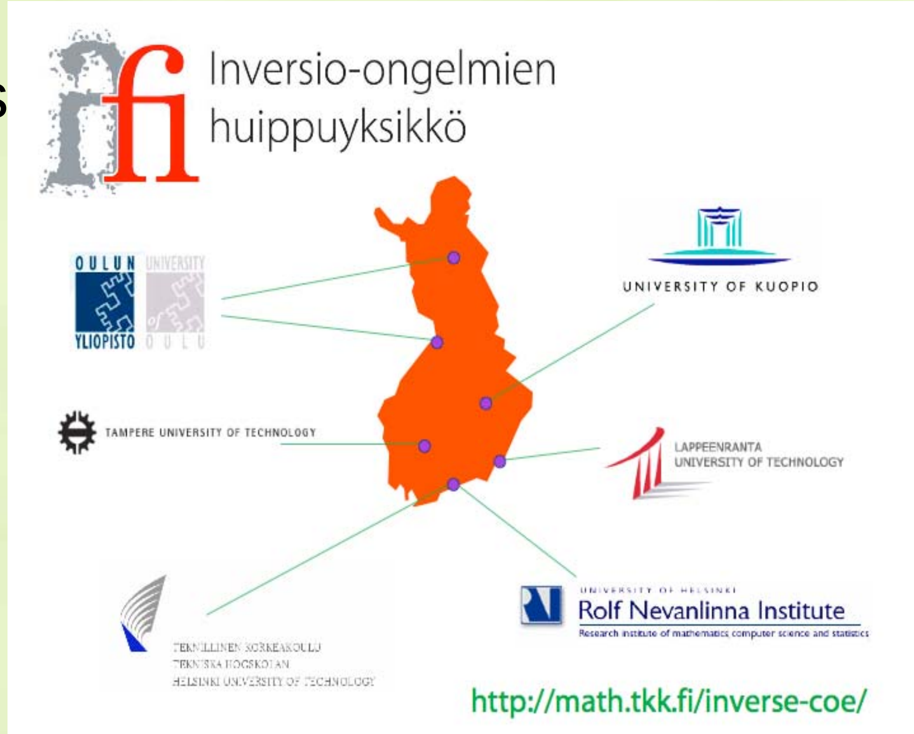
- From consequence to cause: what has caused the observations?
- The observations are explained with a mathematical model
- The data are usually indirect, limited and noisy
- Is the model unique and stable? How much can it vary?
- Much more difficult than the direct problem: mathematical analysis and the search for practical solution methods are essential
- Inverse problems = quest for information

Why is inverse difficult?

- Cf. computing in one's head: compute $15129^{1/2}$. The inverse function is more difficult than the direct one: $123^2=(120+3)^2=15129$.
- Nonuniqueness of solution: Assume the cover of a well and its (convex) aperture in the plane (2D). Which cover shapes never slip through the planar aperture in any orientation (3D)? *Hadamard 2: Uniqueness of the inverse mapping?*
- Errors: Even though the formal inverse solution exists, the smallest error in data can lead to an unbounded error in the solution. *Hadamard 3: Continuity of the inverse mapping?*
- How do we know what is the maximal information derivable from the data, and what is the corresponding mathematical model?

IP-Centre of excellence

- 6 universities, everything from viruses to galaxies, from pure mathematics to industrial applications
- Wide international network
- World leader
- Long experience in mathematical methods in various applications

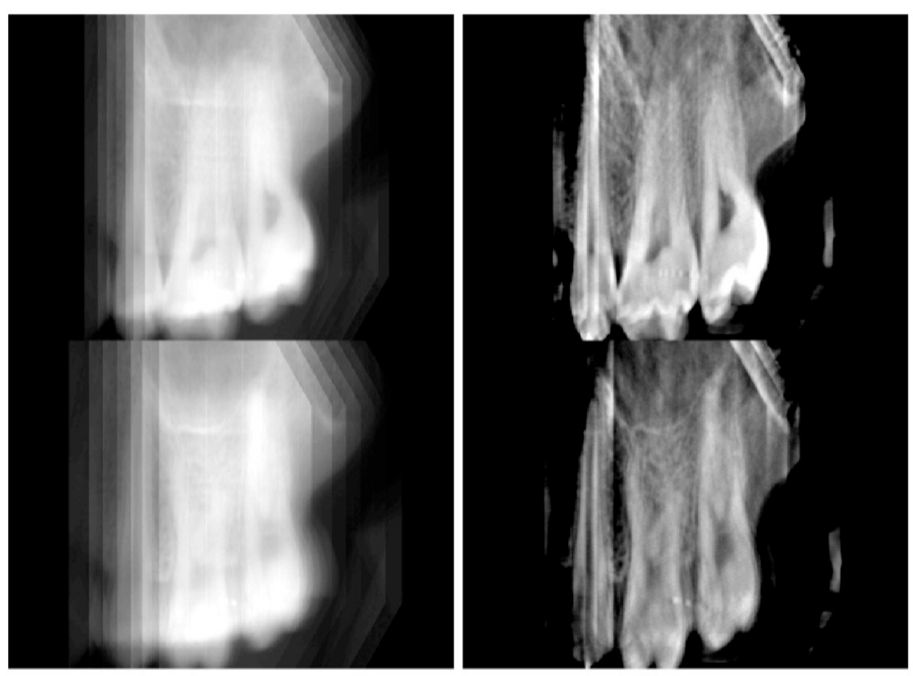


Research areas at TUT

- Space research, dynamical systems, climate/ecological studies (carbon cycle+biomass), remote sensing (laser scanning), biomedical imaging
- Generalized projection operators, phase-space tomography, Poincare inverse problem of dynamics, optimal combination of multiple data modes (maximum compatibility estimate, MCE), information content of data, model selection/reduction/expansion, efficient algorithms for large datasets

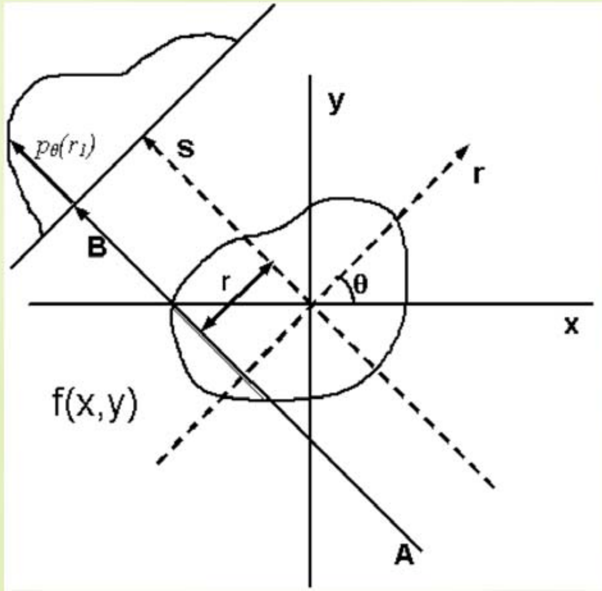
Dental tomography

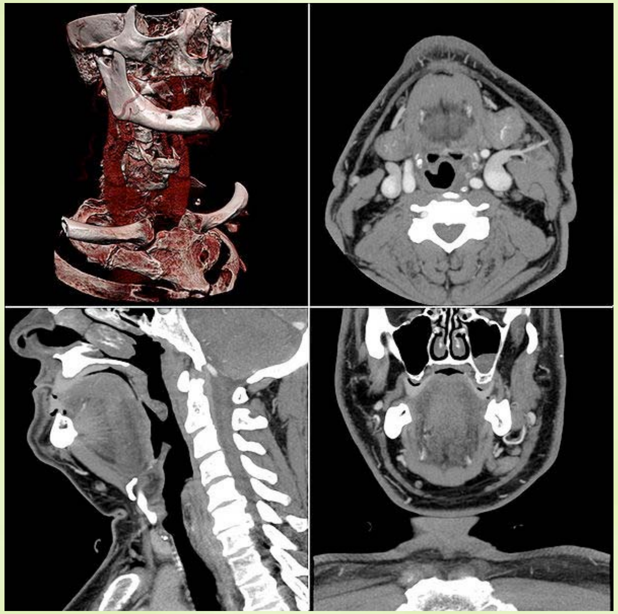
- Limited angle X-ray data
- Complete 3-D model of tooth
- Statistical inversion yields a better solution than the traditional X-ray projection (Bayes: *a posteriori* -distribution includes *a priori* -information)



Biomedical tomography

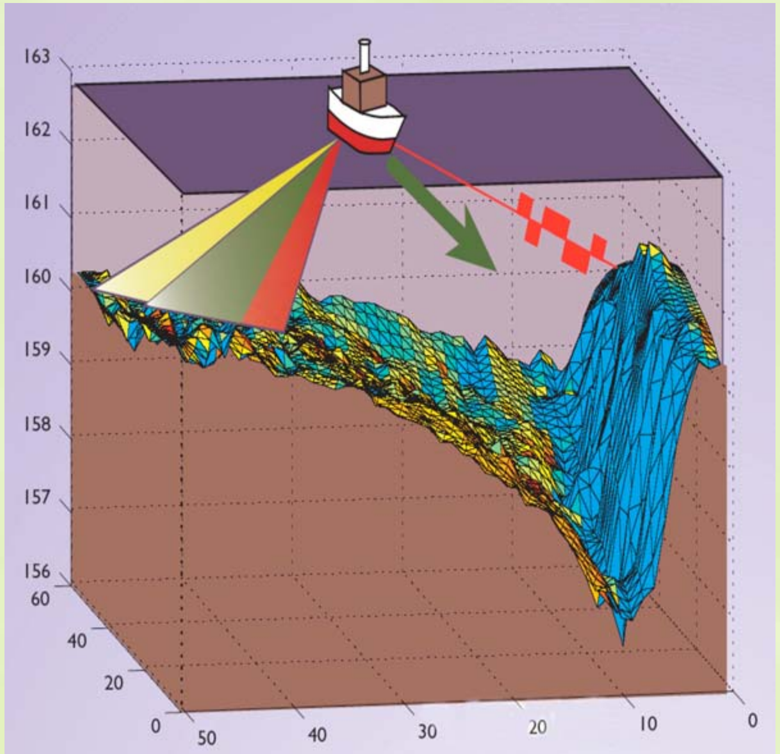
- Radon transform ~100 y ago started the mathematics of imaging (X-ray data)
- Many tomographic methods: MRI, PET, etc.
- New: EIT, diffusion tomography
- EIT is based of the mathematical solution of the *Calderon problem* (2000's): electric conductivity inside the the target is determined from voltage measurements on the boundary
- Breast cancer, pulmonary activity: tissues conduct electricity in different ways





Sonar and radar

- Full 3-D sonar (side directions included)
- Complete topographic model of river bottom
- Pulse coding and inversion methods essential (can even derive *perfect codes*)
- Mathematics makes weather radar work even in difficult conditions



Space radars

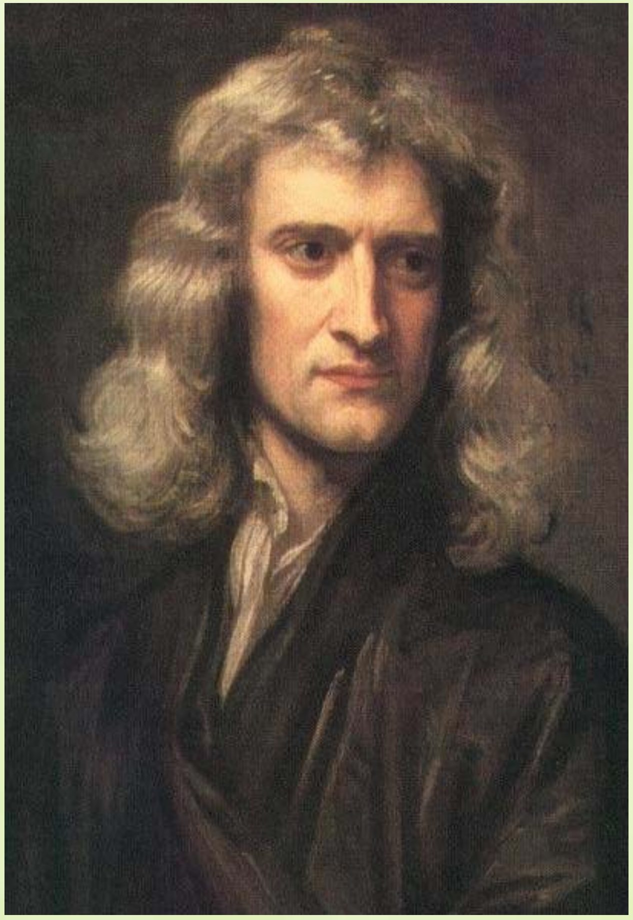
- Sodankylä EISCAT 32 m
- Arecibo 300 m
- Ionosphere, space debris, moon, asteroids

Galaxies and dark matter

- We live at the edge of the disk
- Dust obscures a lot
- But high density of material: lots of data
- How is *dark matter* distributed? What is it made of?
- Thus: what potential field (=> distribution of matter) holds the system together?

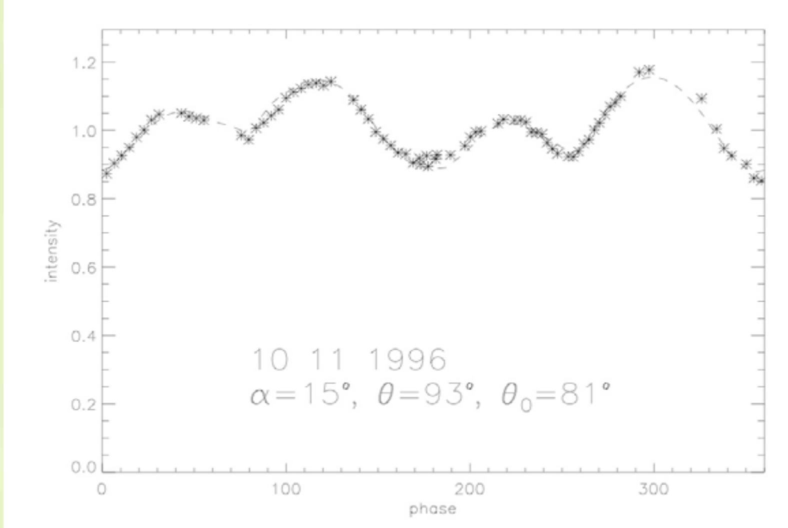
Generalization of Newton's inverse problem

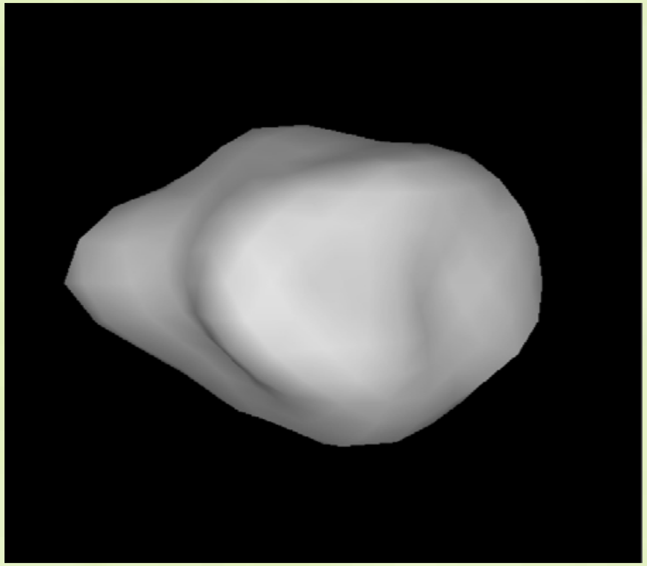
- $F \propto 1/r^2$ => elliptic orbit (conical sections)
- Inverse problem: elliptic orbit => $1/r^2$ (when motion around foci)
- How to generalize this to N>>2 bodies (e.g. positions and velocities of a billion stars)?
- Tomography in six dimensions!
- Poincare IP of dynamics: torus construction



Space research

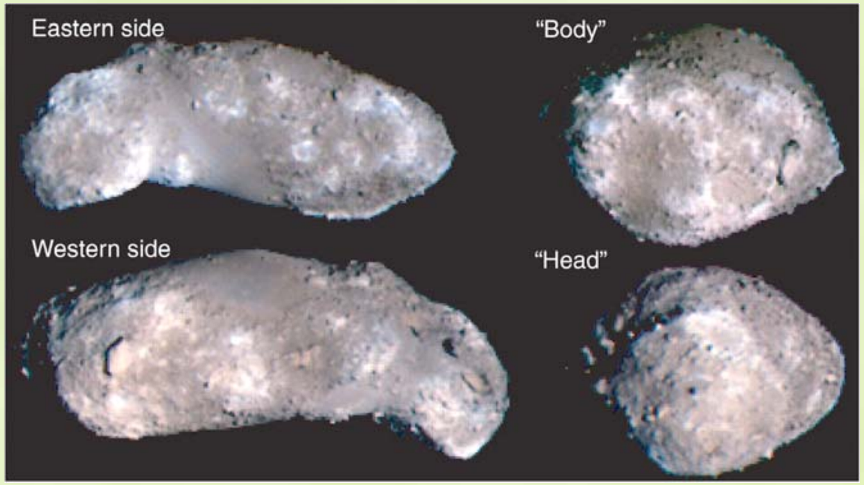
- Brightness measurements (lightcurves) of asteroids at various times (as the target moves in its orbit and rotates)
- What kind of an object created the data?
- Complete model of shape, surface and spin



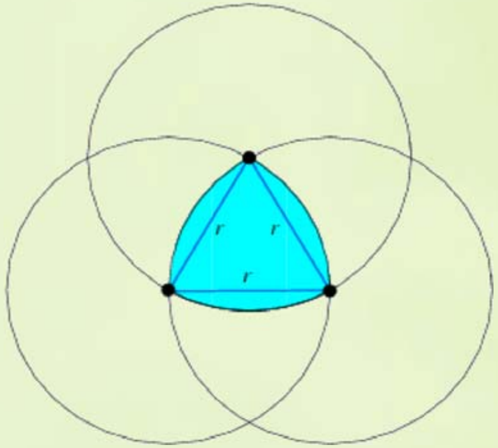


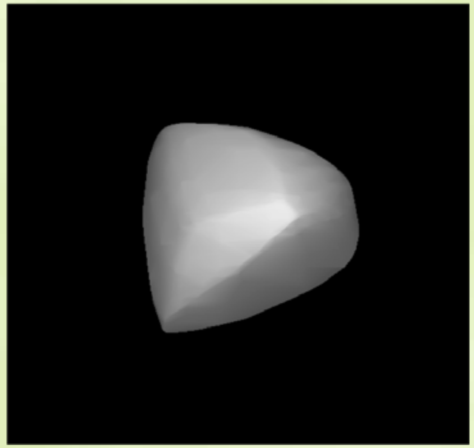
Solar system full of asteroids

- Millions of objects ~1-100 km-class, innumerable smaller ones
- Each a world of its own
- “Jurassic Park”: asteroids tell us about the birth and evolution of the solar system
- Similar populations around other stars
- Direct images only of a few asteroids

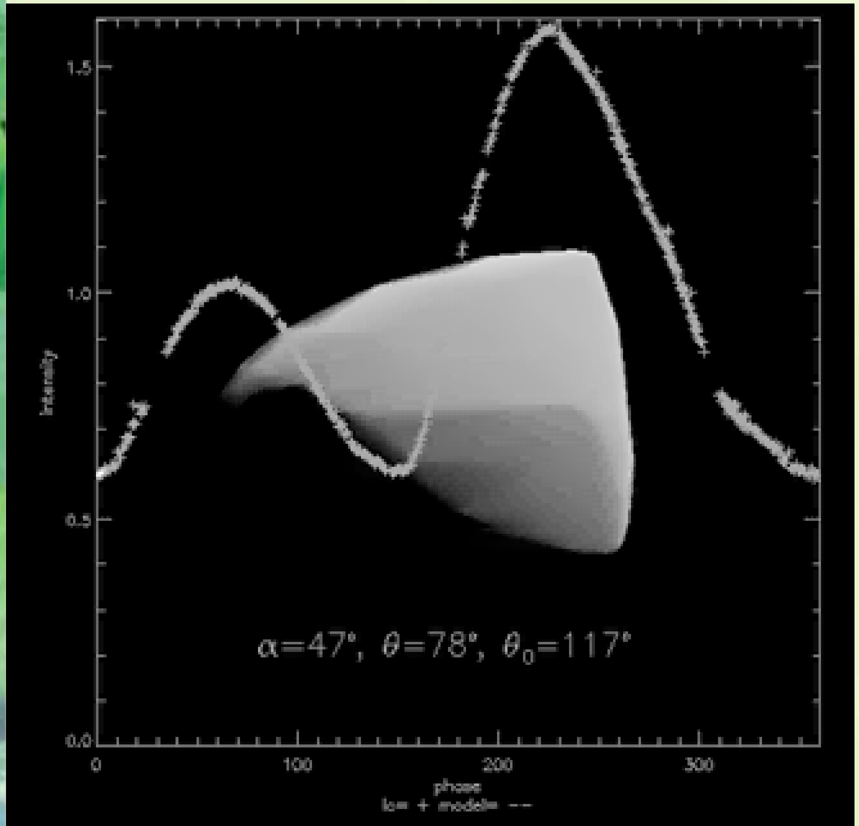


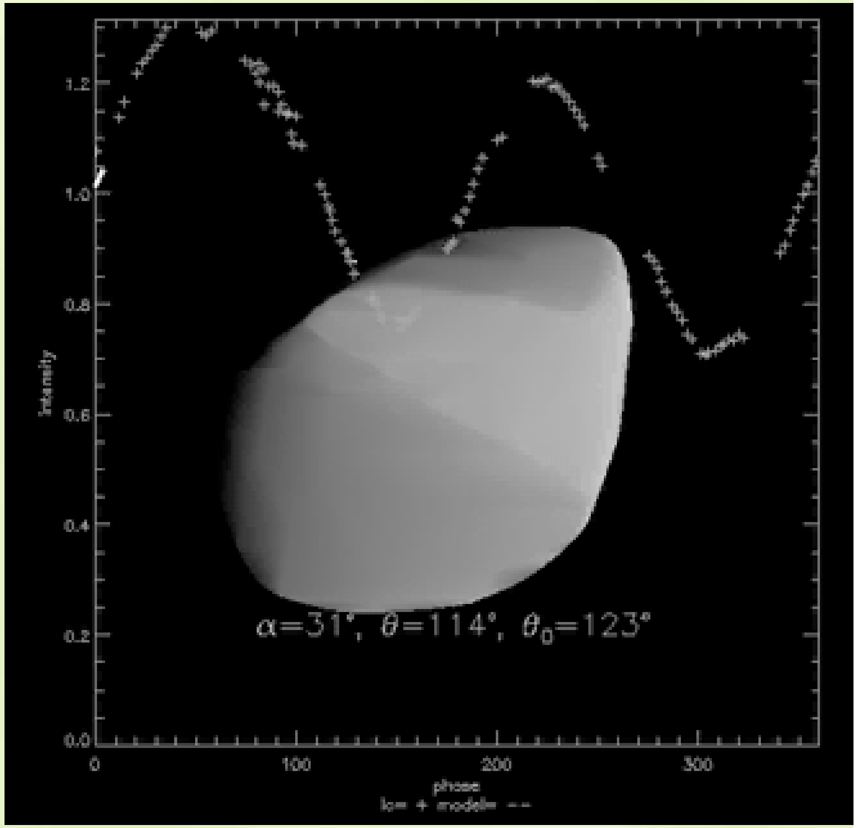
Mathematics maps asteroids





- *Generalized projections:* inverse problem
- Usual areas of projections insufficient (cf. Reuleaux triangle)
- Various viewing and illumination directions yield a both unique and stable solution with convexity constraint
- Tested in practice in several ways
- Was considered unsolvable...

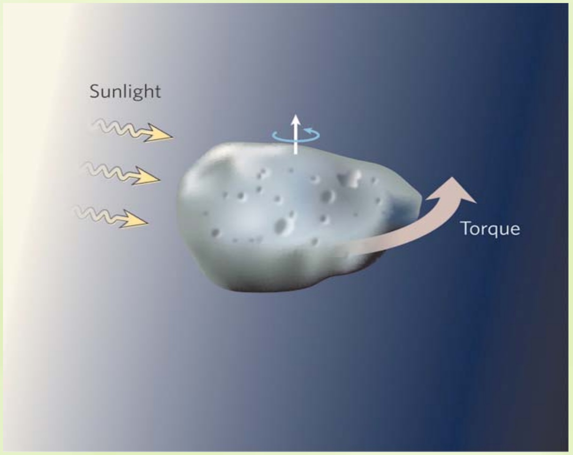


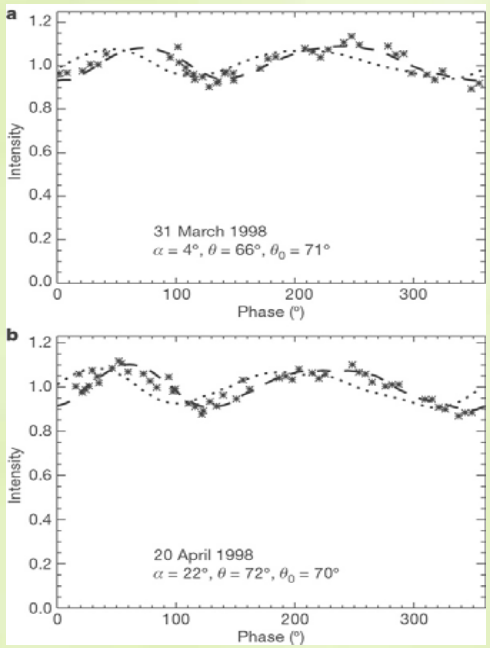


- Reconstruction possible with various observing directions
- Inexpensive data: often even a small telescope and CCD camera (+software) sufficient

New discoveries: Solar motor

- *YORP effect*: solar-accelerated rotation was found from the lightcurves of 1862 Apollo in 2006 [Kaasalainen et al., *Nature* 446, 420 (2007)]
- Several other YORP targets found in the same way after Apollo
- Explains binary asteroids
- “Rubble-pile”asteroid breakup, change of shape; change of orbit (*Yarkovsky-effect*)





“New” phenomenon in solar system

- Sunlight affects the motion of asteroids, including targets delivered into Earth-crossing orbits
- Can even be used for avoiding collisions (let’s paint the asteroid suitably!)

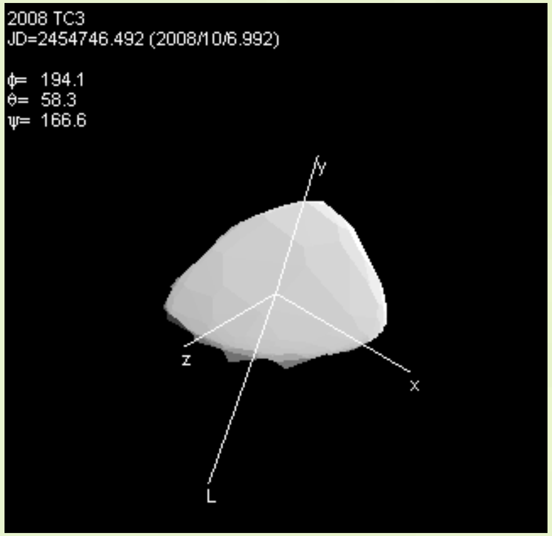

CNN.com
SCIENCE & SPACE
Scientists: Sunlight alters asteroids' spin

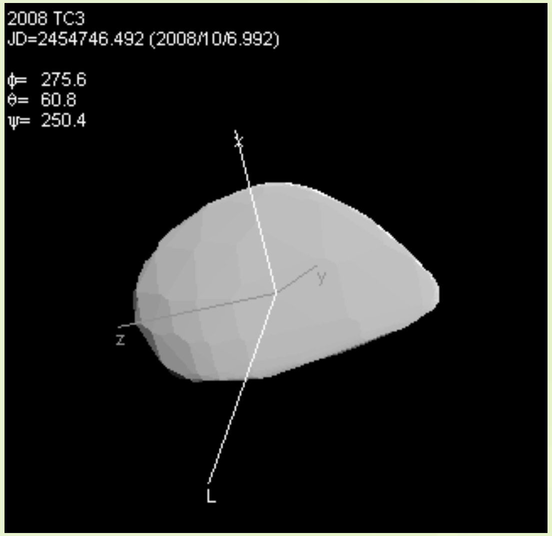

CORRIERE DELLA SERA
SCIENZE E TECNOLOGIE
Asteroidi più pericolosi per la Terra con il «motore» solare naturale


BBC NEWS
Light puts asteroids into a spin


Space & Cosmos
Prediction Proved: Light Speeds Up an Asteroid as it Spins

One that fell to Earth: 2008TC3





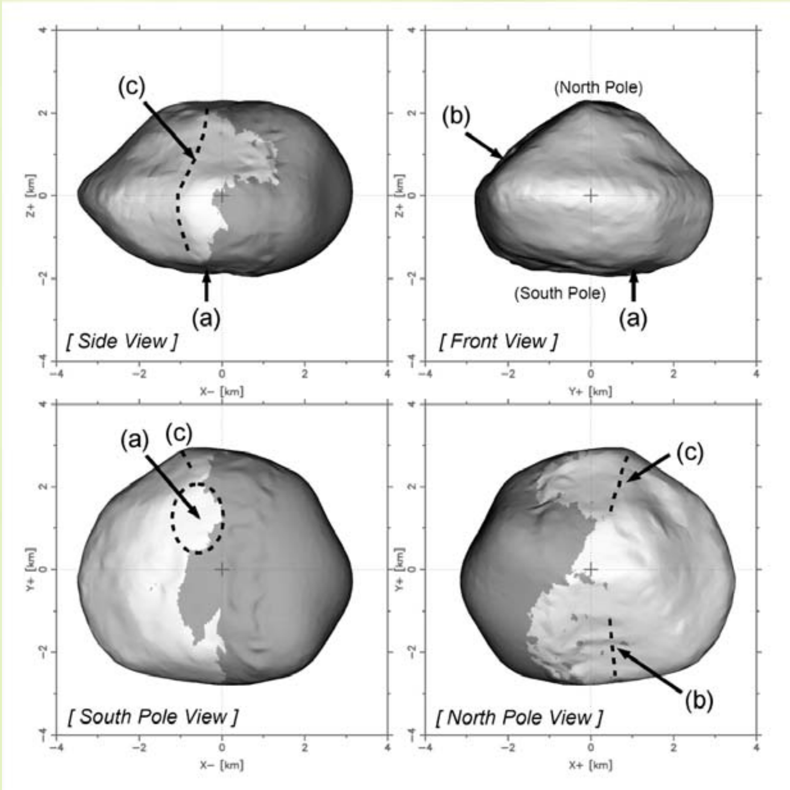
- Small (less than 10 m, mass 100 tons), disintegrated in atmosphere
- Pieces found in Africa, so we know the composition
- And we know the orbit; first predicted collision with Earth...
- Brightness data were obtained just before collision: shape and rotation (precession)

We map the solar system

- N*10 000 mapped asteroids within next 10 years
- The only way of removing the last big “white spot” of the solar system
- Databases and data acquisition international and Internet-based
- Several survey-progs >2010: Pan-STARRS, LSST, Gaia, etc.
- Shape@home screen saver: map your own asteroid!

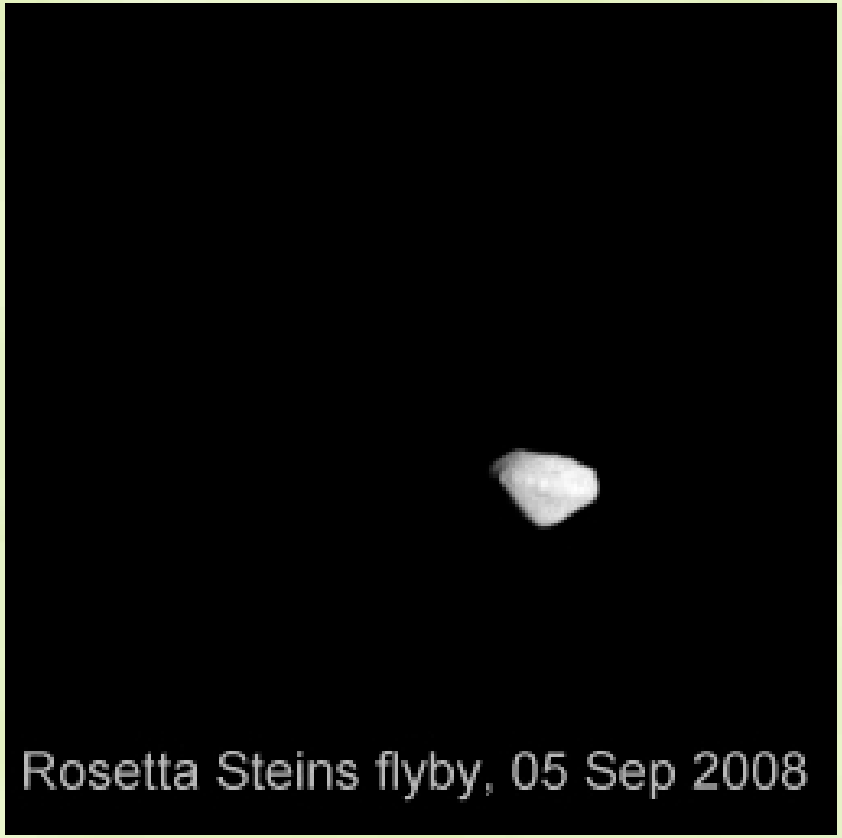
Space research: how to see the dark side

- ESA's Rosetta probe flew fast by the asteroid Steins and saw only one side (and the hi-res camera was lost)
- We reconstructed the other half from brightness data
- Stable solution: strong constraints by images
- YORP effect visible



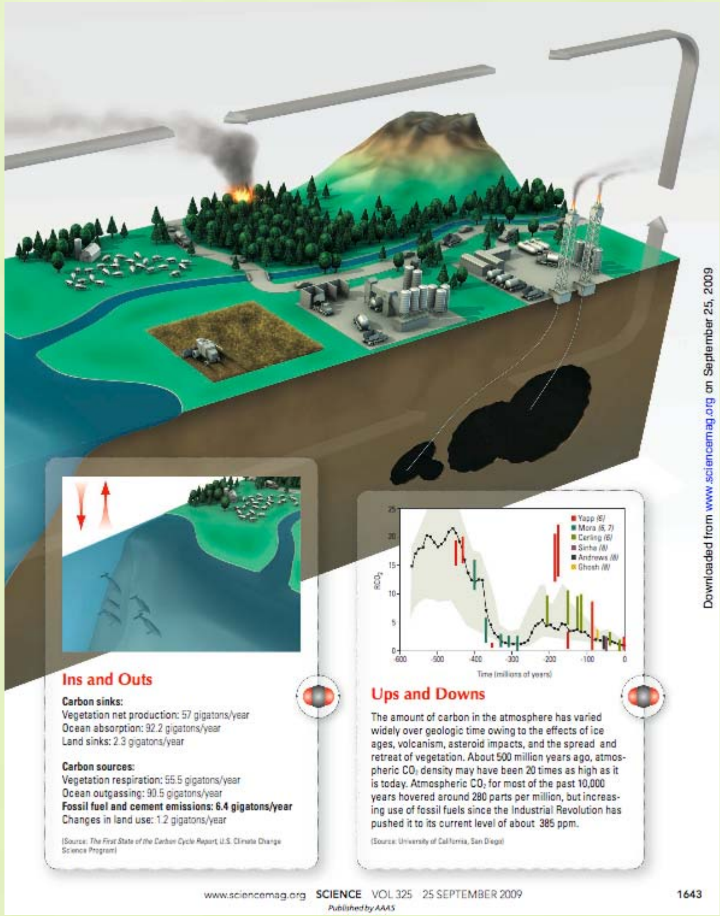
Rosetta: Steins flyby

- Flyby lasted only 7 minutes
- Closest encounter 800 km, target size 5 km
- Speed w.r.t. target 8.6 km/s =31000 km/h
- Dark side reconstruction cost ~0 €, gave significant added value to a project of 700 million €
- Keller et al., Science 2010 (Osiris team, ~50 co-authors)



Carbon cycle, biomass estimation+carbon footprint

- Complex system, model prone to errors
- Model selection problem, design of experiments
- Biomass estimation, disintegration profiles
- Laser scanning technologies, mobile platforms?



Ins and Outs

Carbon sinks:
Vegetation net production: 57 gigatons/year
Ocean absorption: 92.2 gigatons/year
Land sinks: 2.3 gigatons/year

Carbon sources:
Vegetation respiration: 55.5 gigatons/year
Ocean outgassing: 90.5 gigatons/year
Fossil fuel and cement emissions: 6.4 gigatons/year
Changes in land use: 1.2 gigatons/year

(Source: *The First State of the Carbon Cycle Report*, U.S. Climate Change Science Program)

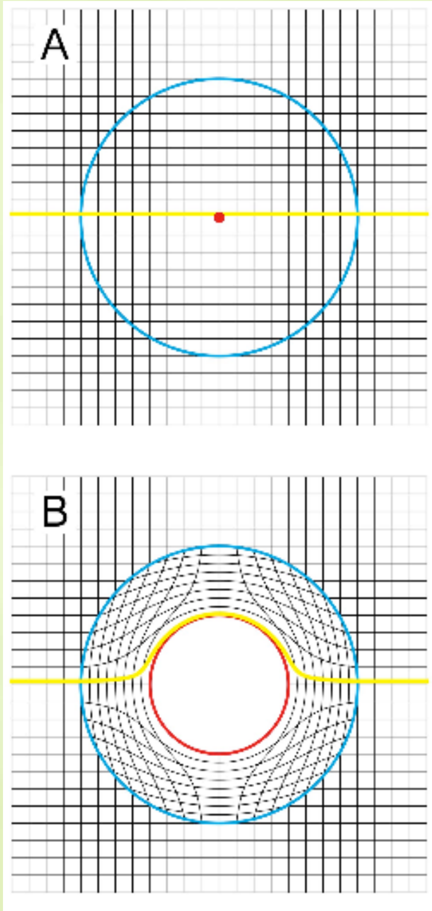
Ups and Downs

The amount of carbon in the atmosphere has varied widely over geologic time owing to the effects of ice ages, volcanism, asteroid impacts, and the spread and retreat of vegetation. About 500 million years ago, atmospheric CO_2 density may have been 20 times as high as it is today. Atmospheric CO_2 for most of the past 10,000 years hovered around 280 parts per million, but increasing use of fossil fuels since the Industrial Revolution has pushed it to its current level of about 385 ppm.

(Source: University of California, San Diego)

Invisibility cloaking

- Light circumvents the target: looks as if nothing obstructed its path
- Inverse problem: what kind of a model of the medium totally removes observations of the target?



Optical wormhole

- Light travels in an invisible tube ("Harry Potter's sleeve"): only the exit is seen
- A way to make, say, a 3-dimensional television
- Metamaterials (negative refraction coeffficient) in the tube/cloak